Improving the performance of linear parallel interference cancellation for CDMA using a method of the statistical mechanics

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In this paper we apply a modified algorithm of the parallel interference cancellation (PIC), recently developed by Tanaka and Okada based on techniques of the statistical neurodynamics [1], to the linear PIC (LPIC) with a tentative soft decision function $f(x) = ax$, and show qualitatively that the modified algorithm enlarges convergent regions of LPIC in terms of the parameter $\beta = K/N$, where K is number of users and N is number of chips per symbol. We call a region in the parameter space of β, in which convergence of the algorithm is guaranteed, simply the convergent region of β.

While LPIC was notorious for the very bad convergence of its algorithm even for comparatively small β, LPIC has been regarded as a practical detector with low computational cost in recent years as it has been shown that its convergence property is improved by using the aid of the partial PIC method of Divsalar et al.[2, 3]. It is necessary to make modifications for improvement of LPIC holding down its computational cost as much as possible. However, in the procedures to determine the partial cancellation coefficients, the trade-off for the increase in speed of convergence of the algorithm is loss of simplicity of computation (increase of computational cost). The remarkable point is that the modified algorithm of PIC of Tanaka and Okada is independent of the partial PIC method and is free from the trade-off relation. Originally, Tanaka and Okada, to begin with, derived time evolution equations of averaged (respect to the random spreading codes and noise of channel) quantities for PIC based on the statistical neurodynamics, and then, using the knowledge of the averaged dynamics of detection, developed a modified algorithm of PIC [1]. Here, let us make clear the settings of our discussion. Under the conditions as the fully-synchronous K-user BPSK-CDMA channel model with additive white Gaussian noise ($n^\mu \sim N(0,1)$ for $\mu = 1, \cdots, N$) with perfect power control, the received signal at chip interval μ is $y^\mu = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} s_k^\mu b_k + \sigma_0 n^\mu$, where $b_k \in \{-1, 1\}$ and $\{s_k^\mu;\ \mu = 1, \cdots, N\}$ are the BPSK information symbol and the random spreading code of user k, respectively, in the similar way as in [3]. The application of Tanaka and Okada's modified algorithm to LPIC is made as follows.

$$x_k^{t+1} = a(h_k - \sum_{l \neq k} W_{kl} x_l^{t-1} - \Gamma_k^t), \tag{1}$$

$$\Gamma_k^{t+1} = \beta a(x_k^t - \Gamma_k^t), \tag{2}$$

where $h_k = N^{-1/2} \sum_{\mu=1}^{N} s_k^\mu y^\mu$, $W_{kl} = \frac{1}{N} \sum_{\mu=1}^{N} s_k^\mu s_l^\mu$ $(k \neq l)$, and Γ_k^t is a term whose convergent value coincide with the Onsager reaction term apeareing in SCSNA analysis [4]. In this paper we limit the discussion to LPIC which converges to MMSE. For LPIC this indicates to take $a = \frac{1}{1+\sigma_0^2}$. However this correspondence breakes down in the modified LPIC (mLPIC) due of modification of the algorithm. Therefore, we need to find a corrected parameter $a = a^*$ which enables mLPIC to converge to the MMSE detector. This can be done by finding a value $a = a^*$ which maximize the convergent value of the signal to interference ratio (SIR).The equilibrium value of SIR is $\frac{1-\beta m^2}{\sigma_0^2 + (1-m)^2}$ with $m = \frac{a}{1+a\beta}$. Thus, from conditions $d\mathrm{SIR}(a^*)/da = 0$ and $d^2\mathrm{SIR}(a^*)/da^2 < 0$ we obtain the corrected parameter $a^* = \frac{\beta - 1 - \sigma_0^2 + \sqrt{4\beta\sigma_0^2 + (\beta - 1 - \sigma_0^2)^2}}{2\beta\sigma_0^2}$ of the optimum mLPIC with which mLPIC has the best performance within the framework. The convergent region of LPIC which converges to MMSE detector is

$$0 < \beta < \beta_c^{\mathrm{LPIC}} \equiv 3 + \sigma_0^2 - 2\sqrt{2 + \sigma_0^2}. \tag{3}$$

On the other hand the convergent condition of the mLPIC is

$$|a^*(\lambda + \alpha)| < 1, \tag{4}$$

where λ is a eigenvalue of the off-diagonal correlation matrix W and $\alpha = (\beta - \lambda)/2 + \sqrt{(\beta - \lambda)^2 - 4\beta}/2$. With the random matrix theory we obtain the convergent region of β as follows.

$$0 \leq \beta < \beta_c^{\mathrm{mLPIC}}(\sigma_0), \tag{5}$$

where $\sqrt{\beta_c^{\mathrm{mLPIC}}(\sigma_0)}$ is the only positive solution of the equation $-2x^3 - 3x^2 + 2\sigma_0^2 x + 1 + \sigma_0^2 = 0$. Figure 1 indicates that mLPIC enlarges convergent regions of β

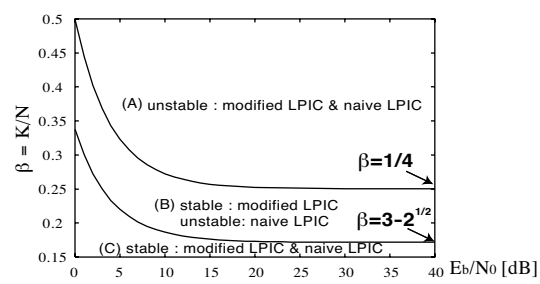

Fig. 1: The convergent region of β versus signal to noise ratio $E_b/N_0 = (2\sigma_0^2)^{-1}$ for LPIC and mLPIC.

References

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