Improving the performance of linear parallel interference cancellation for CDMA using a method of the statistical mechanics

Akira Shojiguchi[1)], Toshiyuki Tanaka[1)], Akira Mizutani[1)], Takumi Mizuno[1)], and Masato Okada[2)]
1) Department of Electronics and Information Engineering, Tokyo Metropolitan University, Tokyo, Japan
2) RIKEN Brain Science Institute, Saitama, Japan, and PREST, JST, Saitama, Japan.

1 Introduction

1.1 Parallel interference cancellation for CDMA

Code-division multiple-access (CDMA) systems allow simultaneous access for multiple users, and it makes possible to identify each user by using a uniquely pre-assigned spreading code [1]. In the CDMA systems one has to cope with the influence of multiple access interference (MAI), since in practice it is typically much harder to maintain orthogonality of the spreading codes. Thus, the focal point for improvement of performance of the CDMA user detection schemes has been to mitigate MAI from received data. One of the most promising methods for practical implementation is multistage parallel interference canceller (PIC) proposed by Varanasi and Aazhang [2]. Since it performs the multiuser detection in a stage-by stage manner, rapid convergence of the stage dynamics towards a detection result is important. In this paper our purpose is to improve the convergence of the detection dynamics of the linear PIC, with the aid of the framework of the statistical neurodynamics [3, 4], a dynamical theory of memory retrieval of the neural associative memory.

1.2 Our problem

We consider the following basic fully-synchronous K-user baseband binary phase-shift-keying (BPSK) CDMA channel model with additive white Gaussian noise $n^\mu \sim N(0,1),\ (\mu = 1, \cdots, N)$ under perfect power control,

$$y^\mu = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} s_k^\mu b_k + \sigma_0 n^\mu, \quad (\mu = 1, \cdots, N) \tag{1}$$

where y^μ is the received signal at chip interval μ, and $b_k \in \{-1, 1\}$ and $\{s_k^\mu;\ \mu = 1, \cdots, N\}$ are BPSK-modulated information symbol and the signature sequece of user k $(k = 1, \cdots, K)$, respectively. The factor $1/\sqrt{N}$ is introduced in order to normalize the power of signal per symbol to 1. In the generic conventional PIC, the tentative decisions at stage t are given as follows:

$$x_k^t = f(u_k^{t-1}) = f(h_k - \sum_{l \neq k} W_{kl} x_l^{t-1}), \tag{2}$$

where h_k is the matched filter output for user k

$$h_k = \frac{1}{\sqrt{N}} \sum_{\mu=1}^{N} s_k^\mu y^\mu. \tag{3}$$

and W_{kl}, $k \neq l$ is the correlation of the spreading codes

$$W_{kl} = \frac{1}{N} \sum_{\mu=1}^{N} s_k^\mu s_l^\mu. \tag{4}$$

We assume the matched filter first stage as the initial decision, which is represented by Eq. (2) as well by assuming $x_k^{-1} \equiv 0$.

In this paper we deal with the linear PIC, for which $f(x) = ax$. Our main purpose in this paper is to improve the performance of the linear PIC with the help of the framework of the statistical neurodynamics [3, 4].

The linear PIC is intensively studied. The reason is as follows. As is discussed in Appendix, by choosing the coefficient a of the decision function $f(x) = ax$, the equilibrium state of the linear PIC can be made equal to the output of the decorrelator and the MMSE detector [8]. This means that we can obtain the decorrelating and MMSE detection results without explicitly inverting the correlation matrix of spreding codes. Since calculation of the inverse of a large matrix is computationally hard, the linear PIC has an advantage over the decorrelator and the MMSE detector in practical point of view. However, the convergence property of the linear PIC is not good: i t may oscillate even when the system load is relatively small.

One of the efforts to circumvent the problem is the partial PIC method originated by Divsalar et al. [9]. It is based on the idea that one should not trust estimates of MAI too much in early stages, because the tentative decisions used to compute the estimates may be less reliable. With such an idea, the partial PIC method provides a parameter to adjust magnitudes of MAI, which we call the partial PIC coefficients. The partial PIC method is reported to converge more rapidly than the conventional one by appropriately choosing the coefficients. Divsalar et al. searched the partial PIC coefficients in a heuristic way. Guo et al. [6, 8] gave a criteria, and provided a systematic way, to obtain the partial PIC coefficients.

We present a different way to improve the performance of the linear PIC. Our approach has an advantage that we do not need other extra subroutines to decide the cancellation coefficients.

2 Density evolution method

Tanaka and Okada utilized an analogical correspondence between demodulating process in PIC for CDMA and memory retrieval process of associative memory and applied the theory of statistical neurodynamics [3, 4] for retrieval dynamics of neural associative memories to the detection dynamics of the PIC for CDMA, to obtain an analytical solution describing the stage dynamics [5].

In this section we apply this theory to the linear PIC and obtain recurrence formulas describing the stage dynamics. In the following discussion we assume the large system limit, that means $K \to \infty$ and $N \to \infty$, while $\beta \equiv K/N$ kept finite. We introduce the auxialiary variables defined as

$$l_t^{\mu,(k)} \equiv s_k^\mu \left(y^\mu - \frac{1}{\sqrt{N}} \sum_{l \neq k}^{K} s_l^\mu x_l^t \right), \tag{5}$$

and make an assumption that $\{l_t^{\mu,(k)}; t = 1, \cdots, T\}$ are random variables following joint Gaussian distributions. Note that we do not ignore their correlations between arbitrary stages. With these variables we can write

$$u_k^t = \frac{1}{\sqrt{N}} \sum_{\mu=1}^{N} l_t^{\mu,(k)}, \tag{6}$$

Therefore, $\{u_k^t\}$ also obey the joint Gaussian distributions. Let the expectation (with respect to the random spreading codes) of $l_t^{\mu,(k)}$ as $B_t^{(k)} \equiv \sqrt{N} E[l_t^{\mu,(k)}]$. Following the statistical neurodynamical theory [4, 5], we obtain the recursion formula for $B_t^{(k)}$ as follows

$$B_t^{(k)} = 1 - \beta U_t (B_{t-1}^{(k)} - x_k^{t-1}), \tag{7}$$

which means that the expectation at stage t depends on the tentative decision x_k^{t-1} at the previous stage. Here, we note that the expectation value of randaom variable $l_t^{\mu,(k)}$ should be independent of μ because all the chips are statistically equivalent. We then decompose the expectation into a user-independent term and the user-dependent term, as follows:

$$B_t^{(k)} = B_t + \delta B_t^{(k)}. \tag{8}$$

Accordingly the recursion equation (7) is decomposed as

$$B_t = 1 - \beta U_t B_{t-1}, \tag{9}$$

$$\delta B_t^{(k)} = -\beta U_t (\delta B_{t-1}^{(k)} - x_k^{t-1}). \tag{10}$$

Using Eq. (10) as well as $\delta B_0^{(k)} = 0$ we can calculate $\delta B_t^{(k)}$ recursively. The user-dependent bias term $\delta B_t^{(k)}$ corresponds to the so-called Onsager reaction term in the field of statistical mechanics.

In the large system limit, surviving macroscopic parameters are as follows:

$$B_t = E[u_k^t], \quad C_{t,s} = E[l_t^{\mu,(k)} l_s^{\mu,(k)}], \tag{11}$$

$$M_t = \tfrac{1}{K} \textstyle\sum_{k=1}^{K} x_k^t, \quad q_{ts} = \tfrac{1}{K} \textstyle\sum_{k=1}^{K} x_k^t x_k^s, \tag{12}$$

$$U_t = \tfrac{1}{K} \textstyle\sum_{k=1}^{K} f'(u_k^{t-1}), \tag{13}$$

Finally the bit error rate is

$$BER_t = Q\left(\frac{B_{t-1}}{\sqrt{C_{tt}}}\right), \tag{14}$$

where $Q(x) = \int_x^\infty Dx$ and $Dx = (1/\sqrt{2\pi}) \exp(-x^2/2)\, dx$. By the theory of statistical neurodynamics, we obtain, by temporally ignoring the user-dependent bias, the following time evolution equations for the macroscopic quantities,

$$B_t = 1 - \beta U_t B_{t-1}, \tag{15}$$

$$\begin{aligned} C_{ts} = {} & \sigma_0^2 + \beta(1 - M_t - M_s + q_{ts}) + \beta^2 U_t U_s C_{t-1,s-1} \\ & + \sum_{\lambda=0}^{t-1} \prod_{\kappa=\lambda+1}^{t} (-\beta U_\kappa)[\sigma_0^2 + \beta(1 - M_\lambda - M_s + q_{\lambda s})] \\ & + \sum_{\lambda=0}^{s-1} \prod_{\kappa=\lambda+1}^{s} (-\beta U_\kappa)[\sigma_0^2 + \beta(1 - M_\lambda - M_t + q_{\lambda t})], \end{aligned} \tag{16}$$

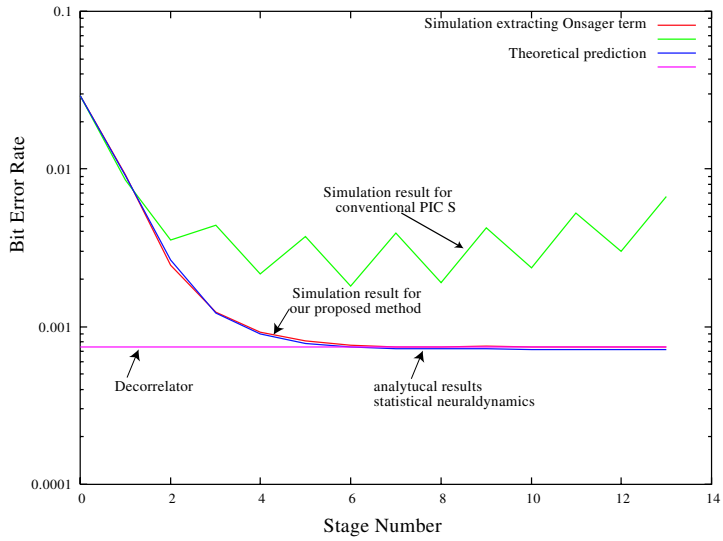

Figure 1: Stage dynamics of linear PICs for $a = 1$. Result of simulation with subtracting the Onsager reaction terms in algorithm ($K = 400$), that of conventional ($K = 400$) simulation, and that predicted by the statistical neurodynamical theory. $\beta = 0.2$, signal to noise ratio $E_b/N_0 = 8$ [dB]. The bit error rate of the decorrelator is also shown as a reference.

$$M_{t+1} = \int f(B_t + \sqrt{C_{tt}}z)Dz, \tag{17}$$

$$U_{t+1} = \frac{1}{\sqrt{C_{tt}}} \int z f(B_t + \sqrt{C_{tt}}z)Dz, \tag{18}$$

$$\begin{aligned} q_{t+1,s+1} = & \int\int\int f(B_t + \sqrt{C_{ts}}z + \sqrt{C_{tt} - C_{ts}}u) \\ & f(B_s + \sqrt{C_{ts}}z + \sqrt{C_{ss} - C_{ts}}v)\, Dz\, Du\, Dv, \end{aligned} \tag{19}$$

where initialization should be done with B_{-1} and $C_{-1-1} = \sigma_0^2 + \beta$.

So far, we have ignored in the analysis the user-dependent bias. Preliminary numerical experiments on the linear PIC revealed, however, that the theory gives a rather poor prediction to the detection dynamics of the linear PIC. Based on the observation, we make the working assumption that the poor predictability of the theory is caused by the absence of the user-dependent bias, and propose a modified linear PIC algorithm, in which the user-dependent bias terms are algorithmically subtracted. By doing the subtraction one can expect that the detection dynamics of the proposed linear PIC algorithm is well predicted by the theory. The proposed algorithm reads

$$x_k^t = f(u_k^{t-1} - \delta B_{t-1}^{(k)}) = a[u_k^{t-1} - \delta B_{t-1}^{(k)}], \tag{20}$$

where definition of u_k^{t-1} is in Eq. (3) and the recurence formula of $\delta B_{t-1}^{(k)}$ is given by Eq. (10) as well as $\delta B_0^{(k)} = 0$.

3 Numerical result and conclusion

In this section we examine the solutions of theretical results and compare them with the numerical simulations on the proposed linear PIC algorithm. Figure 1 shows a comparison between simulation results of the modified PIC ($f(x) = x$)

and the theoretical prediction for the case of $a = 1$. Simulation results of the conventional PIC and those of the decorrelator are also shown for comparison. The theory and the simulation result on the proposed linear PIC coincide with each other. A remarkable point is that the convergence speed of the linear PIC was significantly improved by subtracting the user-dependent bias terms.

We have demonstrated that the subtraction of the user-dependent bias is effective in improving convergence property of the linear PIC. It should be mentioned, however, that the subtraction of the user-dependent bias in the linear PIC is formally similar to applying to the linear PIC the original proposal of the partial PIC by Divsalar et al., because the user-dependent bias in the linear case is proportional to the decision statistics in the previous stage. The advantage of the proposed linear PIC algorithm is its simplicity: The required compuational cost for additional computation is almost negligible, and one does not even have to determine values of additional parameters, such as the partial PIC coefficients, either heuristically or analytically.

We have discussed in this paper the case of $a = 1$, where the equilibrium of the linear PIC is equivalent to the output of decorrelator. From the discussion in appendix, it is straightforward to extend the proposed method for application to the linear PIC whose equilibrium corresponds to the output of the MMSE detector.

Appendix: Relation between linear PIC, decorrelator, and MMSE detector

In this appendix we discuss the relation between the linear PIC, the decorrelator, and the MMSE detector. The stage dynamics of the linear PIC with $f(x) = ax$ is obtained as follows:

$$\begin{aligned}
\boldsymbol{u}^0 &= \boldsymbol{h} && \text{(A.1)}\\
\hat{\boldsymbol{b}}^0 &= a\boldsymbol{u}^0 = a\boldsymbol{h} && \text{(A.2)}\\
\boldsymbol{u}^1 &= \boldsymbol{h} - W\boldsymbol{b}^0 \\
&= \boldsymbol{h} - aW\boldsymbol{h} = (I - aW)\boldsymbol{h} && \text{(A.3)}\\
\hat{\boldsymbol{b}}^1 &= a\boldsymbol{u}^1 = a(I - aW)\boldsymbol{h} && \text{(A.4)}\\
&\cdots \\
\boldsymbol{u}^t &= (I - aW + a^2W^2 - \cdots + (-a)^tW^t)\boldsymbol{h} \equiv A^t\boldsymbol{h}, && \text{(A.5)}\\
\boldsymbol{b}^{t+1} &= a\boldsymbol{u}^t = aA^t\boldsymbol{h} && \text{(A.6)}
\end{aligned}$$

where $\boldsymbol{h}$ represents the matched filter output vector given in Eq. (3). We let $W \equiv R - I$, where R is the correlation matrix of the spreading codes defined by

$$R_{ij} = \frac{1}{N}\sum_{\mu=1}^{N} s_i^\mu s_j^\mu. \tag{A.7}$$

If the following limit exists,

$$\lim_{t\to\infty} A^t = (I + aW)^{-1}, \tag{A.8}$$

we obtain a convergent solution for the linear PIC as

$$\hat{\boldsymbol{b}}^\infty = \mathrm{sign}[a(I + aW)^{-1}\boldsymbol{h}]. \tag{A.9}$$

In this paper we dealt with the case of $a = 1$. As was seen in Fig. 1, both the theoretical prediction and the improved stage dynamics of bit error rate converged to that of the decorrelator. This fact is understood as follows. Since $I + W = R$, the equilibrium state of the linear PIC with $a = 1$ corresponds to the output of the decorrelator, as follows

$$\hat{\boldsymbol{b}}^{\text{decorr}} = \text{sign}[R^{-1}\boldsymbol{h}]. \tag{A.10}$$

The MMSE detection is given by

$$\hat{\boldsymbol{b}}^{\text{MMSE}} = \text{sign}[(R + \sigma^2 I)^{-1}\boldsymbol{h}]. \tag{A.11}$$

Therefore, If we choose a for the linear PIC as

$$a = \frac{1}{1 + \sigma^2}, \tag{A.12}$$

the equilibrium state of the linear PIC corresponds to the output of the MMSE detector, because,

$$R + \sigma^2 I = (1 + \sigma^2)I + W. \tag{A.13}$$

References

[1] S. Verdú, *Multiuser Detection*, Cambridge, Cambridge Univ. Press, 1998.

[2] M. K. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications", *IEEE Trans. Comm.*, vol. 38, pp. 509–519, 1990.

[3] S. Amari and K. Maginu, "Statistical neurodynamics of associative memory" *Neural Networks*, vol. 1, pp. 63–73, 1988.

[4] M. Okada, "A hierarchy of macrodynamical eqations for associative memory", *Neural Networks*, vol. 8, pp. 833–838, 1995.

[5] T. Tanaka and M. Okada, "Approximate belief propagation, density evolution, and statistical neurodynamics for CDMA multiuser detection", submitted to *IEEE Trans. Inform. Theory*, 2003.

[6] D. Guo, L. K. Rasmussen, and T. J. Lim, "Linear parallel interference cancellation in long-code CDMA multiuse detection", *IEEE J. Select. Areas Comm.*, vol. 17, pp. 2074–2081, 1999.

[7] L. K. Rasmussen and J. Oppermann, "Ping-pong effects in Linear parallel interference cancellation for CDMA", *IEEE Trans. Wire. Comm.*, vol. 2, pp. 357–363, 2003.

[8] D. Guo, L. K. Rasmussen, S. Sun, and T. J. Lim, "A matrix-algebraic approach to linear parallel interference cancellation in CDMA", *IEEE Trans. Comm.*, vol. 48, pp. 1–9, 2000.

[9] D. Divsalar, M. K. Simon, and D. Raphaeli, "Improved parallel interference cancellation for CDMA" *IEEE Trans. Comm.*, vol. 46, pp. 258–268, 1998.